

But he will be excellently prepared to reach for more extensive treatises that include such helpful essentials.

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1. N. I. AKHIEZER, *Theory of Approximation*, Ungar, New York, 1956.
2. P. L. BUTZER & H. BERENS, *Semigroups of Operators and Approximation*, Die Grundlehren der math. Wissenschaften, Band 145, Springer-Verlag, New York, 1967.
3. H. S. SHAPIRO, *Topics in Approximation*, Lecture Notes in Math., vol. 187, Springer-Verlag, New York and Berlin, 1971.

3 [2.05, 2.25, 2.40, 6, 7].—A. O. GEL'FOND, *Calculus of Finite Differences* (authorized English translation of the third Russian edition), Hindustan Publishing Corp., Delhi-7, India, 1971, vi + 451 pp., 23 cm. Price \$10.00.

The first edition of this important treatise was published in 1952. A second, revised and enlarged, edition appeared in 1959, and the third edition (which is essentially identical with the second) in 1967, a year before the author's death. The book has been translated into several languages, including German, French, Chinese, Czechoslovakian, and Romanian. This appears to be the first translation into English. (For a review of the French edition, see Review 3, this Journal, v. 18, 1964, p. 514.)

The calculus of finite differences relates to three broad areas of analysis: interpolation and approximation, summation of functions, and difference equations. The present author places emphasis on the first of these areas, devoting to it three chapters, or about two-thirds of the book. Approximation processes in the complex plane receive particular attention.

Chapter I starts out with Lagrange's and Newton's interpolation formulas and some elementary facts on divided differences. The discussion then moves on to a general interpolation problem associated with an infinite triangular array of nodes, and to resulting interpolation series. There is a discussion of best approximation by polynomials, in preparation to a convergence result for the Lagrange interpolation process. Other polynomial approximation processes are studied, both for real and complex domains. In a final section, a general interpolation problem is conceived as a moment problem in the complex plane. Chapter II is concerned with convergence and representation properties of Newton's series. These are special interpolation series; the cases of equidistant, as well as arbitrary, interpolation points are studied in detail. Some number-theoretic applications are also included, e.g., the author's own proof of the transcendence of  $e$  and  $\pi$ . Chapter III, the most advanced and most technical chapter, deals with the problem of constructing an entire function from a denumerable set of data, e.g., from the function values at a sequence of points accumulating at infinity. Problems of this sort do not have unique solutions, but can be treated in a meaningful way by imposing suitable restrictions on the growth of the entire function. They have also bearing on the problem of solving linear differential equations of infinite order with constant coefficients, as is shown at the end of the chapter. The remaining two chapters return to a more elementary level, and to more standard topics, Chapter IV dealing with the problem of summation, Ber-

noulli numbers and Bernoulli polynomials, Euler's summation formula etc., and Chapter V dealing with the theory of linear difference equations, the usual algebraic results as well as the principal results on asymptotics, due to Poincaré and Perron.

While the effort of making this work available to the English-speaking community is commendable, the reader must be warned that the translation is seriously deficient and unreliable. The Russian language being devoid of articles, there are the usual mistakes of choosing a definite article when an indefinite one is called for, and vice versa. More seriously, there are numerous instances of semantic distortion which result in statements often totally incomprehensible. For example, on p. 23 one reads "Denote by  $A$  the identity element, which is taken with a certain number  $A$ ", as compared with the original "Denote by sign  $A$  the value 1 taken with the sign of the number  $A$ "; on p. 65 one reads "This property of the power of  $x$  is known as the complete power of  $x$  in the class of functions . . ." instead of "This property of the powers of  $x$  is called completeness of the powers of  $x$  in the class of functions. . ."; on p. 231, ". . . the great Russian mathematician P. L. Chebyshev" is demoted to ". . . the talented Russian mathematician P. L. Chebyshev"; on pp. 255–256 the reader must unscramble sentences like "Let the domain  $D$  go over in the plane of a complex variable  $w$  when  $w = u(z)$  is mapped onto the simply-connected domain  $D_1$ ". In the face of such blatant distortions and a great many other irregularities of translation, the only advice one can give to a dismayed reader is to double-check with one of the other available translations.

W. G.

4 [3].—NOEL GASTINEL, *Linear Numerical Analysis*, translated from the French, Academic Press, New York, 1971, ix + 341 pp., 23 cm. Price \$15.00.

This is a translation of the author's *Analyse Numérique Linéaire*, published in 1966. The translator (unnamed) has taken a few mild liberties, but no doubt with the author's knowledge and consent. The foreword is abridged. In the original, there are chapters, sections, and some subsections, but in the translation only chapters and sections, and some of the titles are changed. One or two figures are omitted. Some theorems are formally stated and numbered in the translation that are not so stated in the original. Otherwise the translation is faithful.

The book itself is strongly algorithmic. The theory is developed from first principles (vector spaces, matrices, a postulational development of determinants) and proceeds to ALGOL programs. The theory is clearly, but succinctly, developed. There are a number of exercises, both theoretical and algorithmic.

Nearly all the standard methods for inversion, direct and iterative, are described, including some attention to SOR. For eigenvalues and eigenvectors, the coverage is a bit less complete. The chapter opens with a brief discussion of interpolative methods, not recommended, however, unless perhaps a very good initial approximation to a root is known. It is also implied in the original and explicitly stated in the translation that the root must be real, which is not strictly true.

After this, which is more or less an aside, the chapter continues with Krylov, Leverrier and Souriau's improvement, Samuelsen, "partitioning" (Bryan), Dan-